K01 SPEED AND VELOCITY

SPH4U

CH 1 (THE BIG PICTURE)

- the linear motion of objects in horizontal, vertical, and inclined planes
- the motion of a projectile in terms of components of its motion
- objects moving in two dimensions
- predict the motion of an object
- technological devices based on the concepts and principles of projectile motion

EQUATIONS

- Speed:

$$v_{av} = \frac{d}{\Delta t}$$

- Velocity:

$$\vec{v}_{av} = \frac{\Delta \vec{d}}{\Delta t}$$

SPEED AND OTHER SCALAR QUANTITIES

- **Kinematics:** the study of motion
- **Scalar Quantity:** quantity that has magnitude, but no direction
- **Instantaneous Speed [m/s]:** speed at a particular instant
- **Average Speed (v_{av}) [m/s]:** total distance of travel divided by total time of travel

$$v_{av} = \frac{d}{\Delta t}$$

SAMPLE PROBLEM 1

At the Molson Indy race in Toronto, Ontario, a driver covers a single-lap distance of 2.90 km at an average speed of 1.50×10^2 km/h. Determine

a) the average speed in metres per second

b) the time in seconds to complete the lap

SAMPLE PROBLEM 1 – SOLUTIONS

a) To convert units, we multiply by factors that are equivalent to 1. We know that 1 km $=$1000 m and 1 h $=$ 3600 s

$$1.50 \times 10^2 \text{ km/h} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 41.7 \text{ m/s}$$

The average speed is 41.7 m/s.

SAMPLE PROBLEM 1 – SOLUTIONS

b) $v_{av} = 41.7 \text{ m/s}$
$d = 2.90 \text{ km} = 2.90 \times 10^3 \text{ m}$
$\Delta t = ?$

Rearranging the equation $v_{av} = \frac{d}{\Delta t}$ to isolate Δt, we have

$$\Delta t = \frac{d}{v_{av}} = \frac{2.90 \times 10^3 \text{ m}}{41.7 \text{ m/s}} = 69.6 \text{ s}$$

The time to complete the lap is 69.6 s.

VELOCITY AND OTHER VECTOR QUANTITIES

- **Vector Quantity:** quantity that has both magnitude and direction
- **Position ($\vec{d}$) [m]:** the distance and direction of an object from a reference point
- **Displacement ($\Delta\vec{d}$) [m]:** change in position of an object in a given direction
  - $\Delta\vec{d} = \vec{d}_2 - \vec{d}_1$

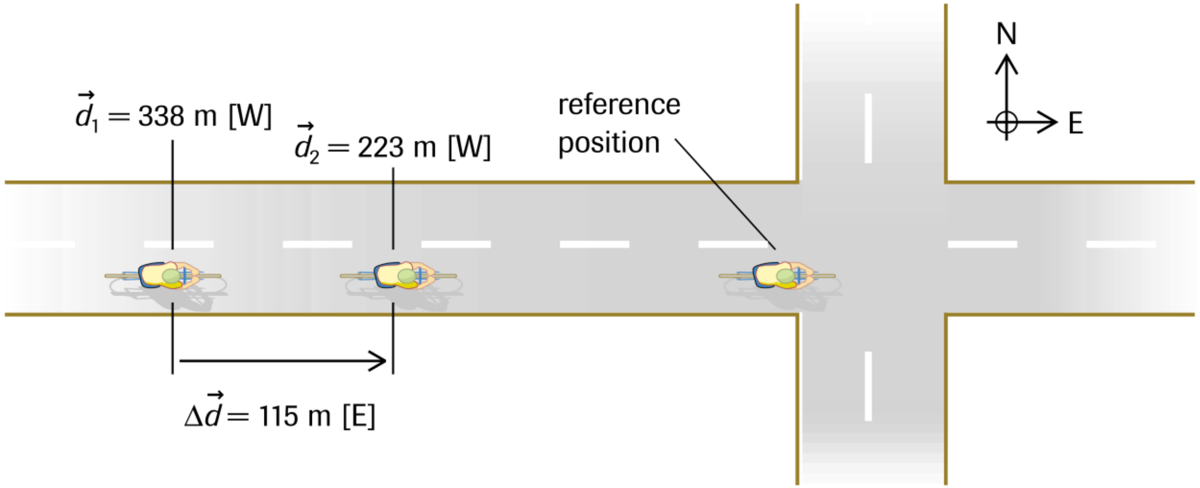


Figure 3
In moving from position $\vec{d}_1$ to $\vec{d}_2$, the cyclist undergoes a displacement $\Delta\vec{d} = \vec{d}_2 - \vec{d}_1$.

VELOCITY AND OTHER VECTOR QUANTITIES

- **Velocity ($\vec{v}$) [m/s]:** the rate of change of position
- **Instantaneous Velocity:** velocity at a particular instant
- **Average Velocity ($\vec{v}_{av}$):** change of position divided by the time interval for that change

$$\vec{v}_{av} = \frac{\Delta \vec{d}}{\Delta t}$$

SAMPLE PROBLEM 2

The cyclist in **Figure 3** takes 25.1 s to cover the displacement of 115 m [E] from $\vec{d}_1$ to $\vec{d}_2$.

a) Calculate the cyclist's average velocity.

b) If the cyclist maintains the same average velocity for 1.00 h, what is the total displacement?

c) If the cyclist turns around at $\vec{d}_2$ and travels to position $\vec{d}_3$ = 565 m [W] in 72.5 s, what is the average velocity for the entire motion?

SAMPLE PROBLEM 2 – SOLUTIONS

(a) $\Delta\vec{d} = 115 \text{ m [E]}$

$\Delta t = 25.1 \text{ s}$

$\vec{v}_{av} = ?$

$$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$$

$$= \frac{115 \text{ m [E]}}{25.1 \text{ s}}$$

$$\vec{v}_{av} = 4.58 \text{ m/s [E]}$$

The cyclist's average velocity is 4.58 m/s [E].

SAMPLE PROBLEM 2 – SOLUTIONS

(b) $\Delta t = 1.00 \text{ h} = 3600 \text{ s}$

$\vec{v}_{av} = 4.58 \text{ m/s [E]}$

$\Delta\vec{d} = ?$

$$\begin{aligned}\Delta\vec{d} &= \vec{v}_{av}\Delta t \\ &= (4.58 \text{ m/s [E]})(3600 \text{ s}) \\ \Delta\vec{d} &= 1.65 \times 10^4 \text{ m [E] or } 16.5 \text{ km [E]}\end{aligned}$$

The total displacement is 16.5 km [E].

SAMPLE PROBLEM 2 – SOLUTIONS

(c)
$$\Delta\vec{d} = \vec{d}_3 - \vec{d}_1$$
$$= 565\text{ m [W]} - 338\text{ m [W]}$$
$$\Delta\vec{d} = 227\text{ m [W]}$$

$$\Delta t = 25.1\text{ s} + 72.5\text{ s} = 97.6\text{ s}$$

$$\vec{v}_{av} = ?$$

$$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$$
$$= \frac{227\text{ m [W]}}{97.6\text{ s}}$$
$$\vec{v}_{av} = 2.33\text{ m/s [W]}$$

The average velocity is 2.33 m/s [W].

POSITION AND VELOCITY GRAPHS UNIFORM MOTION

- Straight line = constant velocity (uniform motion)
- Find velocity by calculating the slope of the line

Table 1 Position-Time Data

Time t (s)	Position $\vec{d}$ (m [S])
0	0
60	330
120	660
180	990

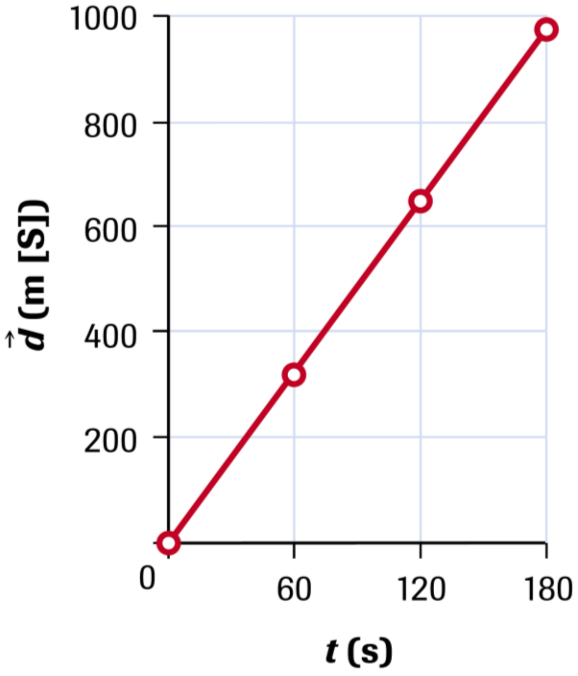


Figure 5
Position-time graph of the runner's motion

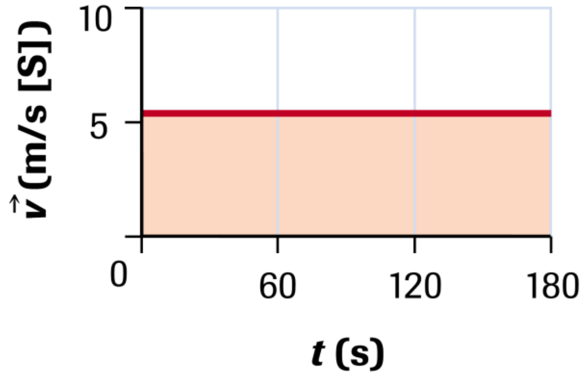


Figure 6
Velocity-time graph of the runner's motion

SAMPLE PROBLEM 3

Describe the motion represented by the position-time graph shown in **Figure 7**, and sketch the corresponding velocity-time graph.

SAMPLE PROBLEM 3 – SOLUTIONS

The slope of the line is constant and it is negative. This means that the velocity is constant in the easterly direction. The initial position is away from the origin and the object is moving toward the origin. The velocity-time graph can be either negative west or positive east as shown in **Figure 8**.

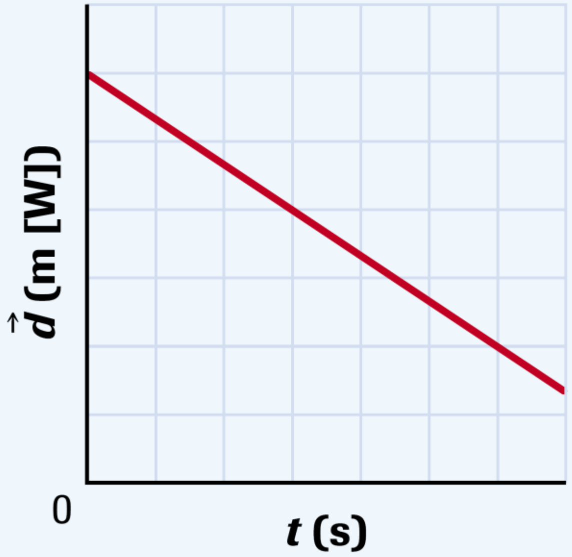


Figure 7
Position-time graph

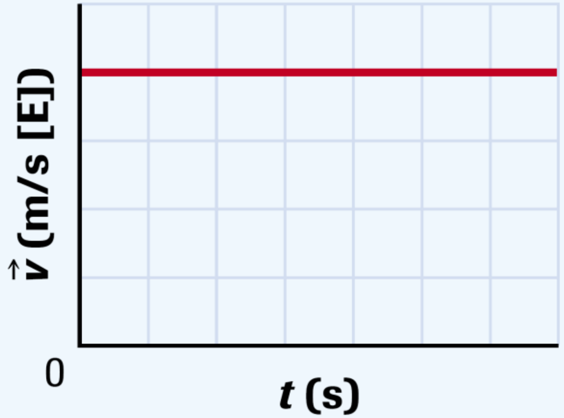


Figure 8
Velocity-time graph

POSITION AND VELOCITY GRAPHS NONUNIFORM MOTION

- Average velocity between any two points:

$$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$$

Table 2 Position-Time Data

Time t (s)	Position $\vec{d}$ (m [fwd])
0	0
2.0	4.0
4.0	16
6.0	36
8.0	64

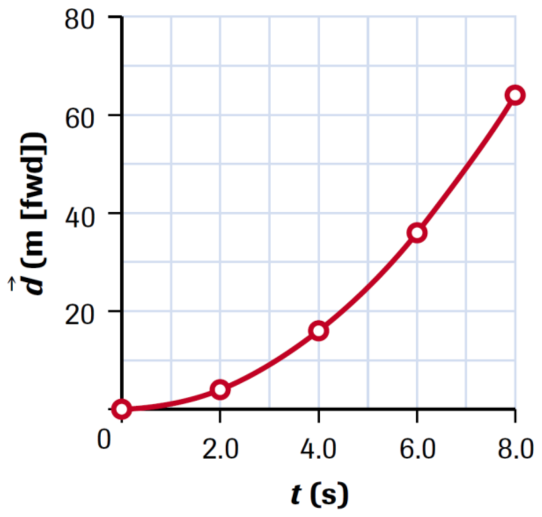


Figure 9
Position-time graph

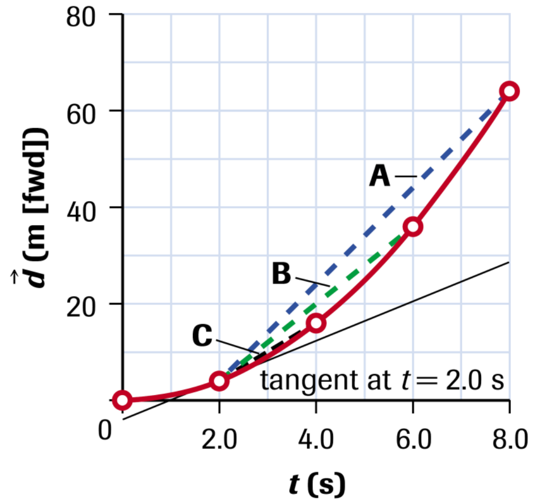


Figure 10

POSITION AND VELOCITY GRAPHS NONUNIFORM MOTION – CONT.

- Instantaneous velocity:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{d}}{\Delta t}$$

- **Tangent:** a straight line that touches a curve at a single point and has the same slope as the curve at that point
- How to draw a tangent

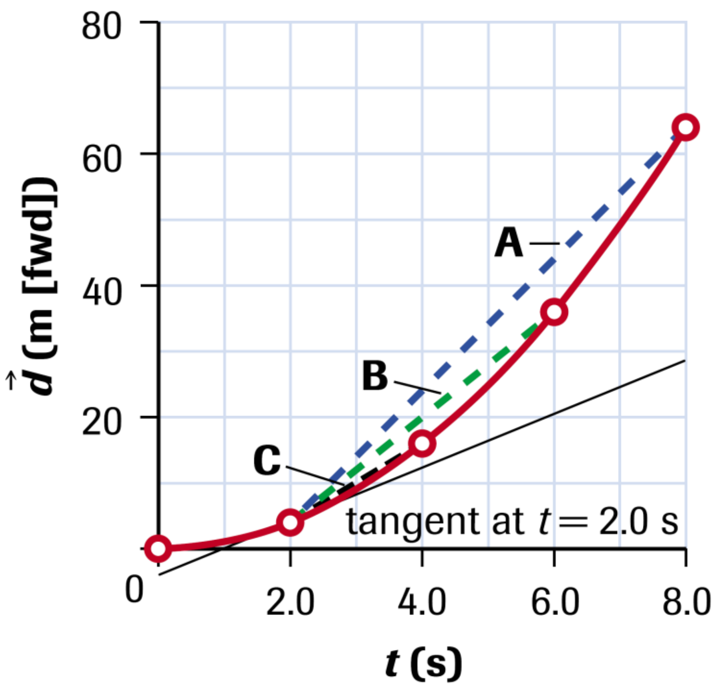


Figure 10

SAMPLE PROBLEM 4

Figure 11 is the position-time graph for a golf ball rolling along a straight trough which slopes downward from east to west. We arbitrarily choose one-dimensional coordinates on which the origin is at the western end of the trough.

(a) Describe the motion.

(b) Calculate the instantaneous velocity at $t = 3.0$ s.

(c) Determine the average velocity between 3.0 s and 6.0 s.

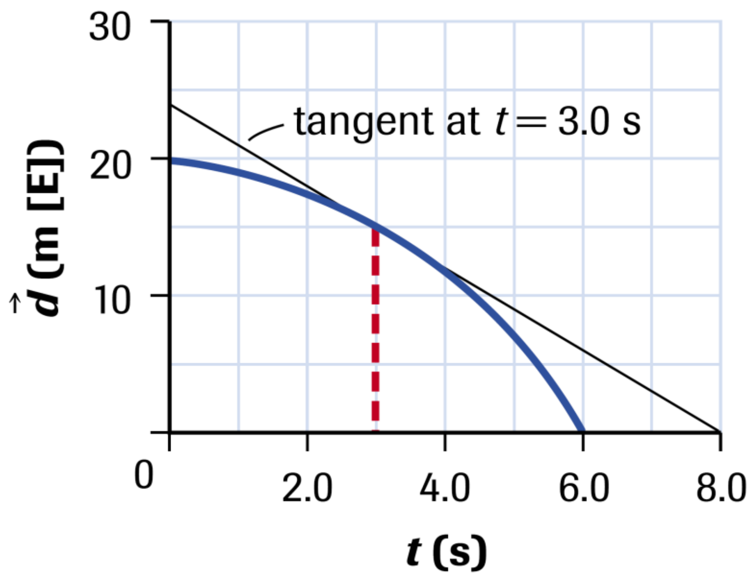


Figure 11
Position-time graph

SAMPLE PROBLEM 4 – SOLUTIONS

(a) The slope is zero at $t = 0.0$ s, then it becomes negative. Thus, the velocity starts off at zero and gradually increases in magnitude in the westerly direction. (Negative east is equivalent to positive west.) The object starts at a position east of the reference point or origin and then moves westward arriving at the origin 6.0 s later.

SAMPLE PROBLEM 4 – SOLUTIONS

(b) The instantaneous velocity at $t = 3.0$ s is the slope of the tangent at that instant. Thus,

$$\begin{aligned}
\vec{v} &= \text{slope} = m = \frac{\Delta\vec{d}}{\Delta t} \\
&= \frac{0.0\text{ m} - 24\text{ m [E]}}{8.0\text{ s} - 0.0\text{ s}} \\
&= -3.0\text{ m/s [E]} \\
\vec{v} &= 3.0\text{ m/s [W]}
\end{aligned}$$

The instantaneous velocity at 3.0 s is approximately 3.0 m/s [W].
(This answer is approximate because of the uncertainty of drawing the tangent.)

SAMPLE PROBLEM 4 – SOLUTIONS

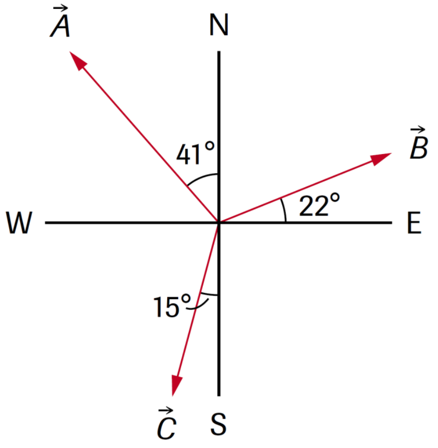
(c) We apply the equation for average velocity:

$$\begin{aligned}
\vec{v}_{\text{av}} &= \frac{\Delta \vec{d}}{\Delta t} \\
&= \frac{0.0\ \text{m} - 15\ \text{m [E]}}{6.0\ \text{s} - 3.0\ \text{s}} \\
&= -5.0\ \text{m/s [E]} \\
\vec{v}_{\text{av}} &= 5.0\ \text{m/s [W]}
\end{aligned}$$

The average velocity between 3.0 s and 6.0 s is 5.0 m/s [W].

DISPLACEMENT AND VELOCITY IN TWO DIMENSIONS

- Use degrees and compass points to describe the direction of the vector



directions of vectors:
$\vec{A}$ [41° W of N]
$\vec{B}$ [22° N of E]
$\vec{C}$ [15° W of S]

Figure 16
Notation for specifying the directions of vectors

- **Total Displacement:** sum of all displacement vectors

$$\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2 + \cdots$$

SAMPLE PROBLEM 5

In 4.4 s, a chickadee flies in a horizontal plane from a fence post (P) to a bush (B) and then to a bird feeder (F), as shown in **Figure 18(a)**. Find the following:

(a) total distance travelled

(b) average speed

(c) total displacement

(d) average velocity

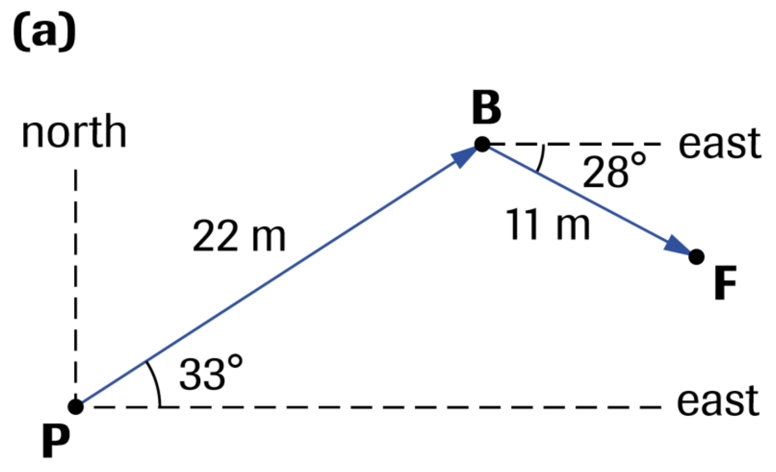


Figure 18
For Sample Problem 5
(a) The chickadee takes 4.4 s to complete the motion shown.

SAMPLE PROBLEM 5 – SOLUTIONS

(a) The total distance travelled is a scalar quantity.

$d = 22\text{ m} + 11\text{ m} = 33\text{ m}$

(b) $d = 33\text{ m}$

$\Delta t = 4.4\text{ s}$

$v_{av} = ?$

$$v_{av} = \frac{d}{\Delta t}$$

$$= \frac{33\text{ m}}{4.4\text{ s}}$$

$$v_{av} = 7.5\text{ m/s}$$

The average speed is 7.5 m/s.

SAMPLE PROBLEM 5 – SOLUTIONS

(c) We will use the method of sine and cosine laws to solve this problem. (Alternatively, we could use the component technique or a vector scale diagram.) We apply the cosine law to find the magnitude of the displacement, $|\Delta\vec{d}|$. From **Figure 18(b)**, the angle B equals 119°.

$$|\Delta\vec{d}_1| = 22\text{ m}$$
$$|\Delta\vec{d}_2| = 11\text{ m}$$
$$\measuredangle B = 119^\circ$$
$$|\Delta\vec{d}| = ?$$

Applying the cosine law:

$$|\Delta\vec{d}|^2 = |\Delta\vec{d}_1|^2 + |\Delta\vec{d}_2|^2 - 2|\Delta\vec{d}_1||\Delta\vec{d}_2|\cos B$$
$$|\Delta\vec{d}|^2 = (22\text{ m})^2 + (11\text{ m})^2 - 2(22\text{ m})(11\text{ m})(\cos 119^\circ)$$
$$|\Delta\vec{d}| = 29\text{ m}$$

SAMPLE PROBLEM 5 – SOLUTIONS

(c) To determine the direction of the displacement, we use the sine law:

$$\frac{\sin P}{|\Delta\vec{d}_2|} = \frac{\sin B}{|\Delta\vec{d}|}$$

$$\sin P = \frac{|\Delta\vec{d}_2| \sin B}{|\Delta\vec{d}|}$$

$$\sin P = \frac{(11\text{ m})(\sin 119^\circ)}{(29\text{ m})}$$

$$\measuredangle P = 19^\circ$$

From the diagram, we see that the direction of the total displacement is $33^\circ - 19^\circ = 14^\circ$ N of E. Therefore, the total displacement is 29 m [14° N of E].

SAMPLE PROBLEM 5 – SOLUTIONS

(d) $\Delta\vec{d} = 29\text{ m }[14° \text{ N of E}]$

$\Delta t = 4.4\text{ s}$

$\vec{v}_{av} = ?$

$$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$$

$$= \frac{29\text{ m }[14° \text{ N of E}]}{4.4\text{ s}}$$

$$\vec{v}_{av} = 6.6\text{ m/s }[14° \text{ N of E}]$$

The average velocity is 6.6 m/s [14° N of E].

SUMMARY

- A scalar quantity has magnitude but no direction.
- Average speed is the total distance travelled divided by the total time of travel.
- A vector quantity has both magnitude and direction.
- Position is the distance with a direction from some reference point.
- Displacement is the change of position.
- Velocity is the rate of change of position.
- Average velocity is change of position divided by the time interval for that change.
- Instantaneous velocity is the velocity at a particular instant.
- Instantaneous speed is the magnitude of the instantaneous velocity.
- The slope of the line on a position-time graph indicates the velocity.
- The area under the line on a velocity-time graph indicates the change of position.
- In two-dimensional motion, the average velocity is the total displacement divided by the time interval for that displacement.

PRACTICE

Readings

- Section 1.1 (pg 6)

Questions

- pg 17 #1-8, 10